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Interacting potential between spinons in the compact QED$_3$ description of the Heisenberg model

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Abstract. We implement a Chern-Simons (CS) contribution into the compact QED_3 description of the antiferromagnetic Heisenberg model in two dimensions at zero temperature. The CS term allows for the conservation of the $SU(2)$ symmetry of the quantum spin system and fixes the flux through a plaquette to be a multiple of π as was shown by Marston. We work out the string tension of the confining potential which acts between the spinons and show that the CS term induces a screening effect on the magnetic field only. The confining potential between spinons is not affected by the CS flux. The strict site-occupation by a single spin 1/2 is enforced by the introduction of an imaginary chemical potential constraint.

PACS. 75.10.Jm Quantized spin models – 11.10.Kk Field theories in dimensions other than four – 11.15.-q Gauge field theories – 11.25.Mj Compactification and four-dimensional models

1 Introduction

Quantum phase transitions of matter near zero temperature have attracted much interests in the recent past. A possible mechanism for high-T_c superconductivity may be a transition between an antiferromagnetic Néel phase and a valence-bond-solid (VBS) phase, see f.i. reference [1]. Frustrated Heisenberg interactions can be mapped into a non-linear sigma model from which it is shown that topological defects play an important role in the spinon deconfinement through the phase transition from a Néel phase to a VBS phase [2]. We shall introduce below gauge theories which cannot predict such kind of phase transitions in non-frustrated Heisenberg models.

At low energy non-frustrated Heisenberg Hamiltonian can be reduced to Dirac actions. Indeed, a gauge field formulation of the antiferromagnetic Heisenberg model in $d = 2$ dimensions leads to a quantum electrodynamic QED_3 action for spinons [12]. It was shown through a renormalization group study of compact (2+1)-dimensionnal Maxwell electrodynamics coupled to fermion field with $SU(N)$ symmetry that the fermions cannot deconfine when N is lower than 20 [23]. This is of peculiar interest for the QED_3 description of the non-frustrated Heisenberg model. Indeed, in the latter case the number of replica is $N = 2$ which implies that the spinons will not deconfine and the Heisenberg model will not present a paramagnetic phase (i.e. no VBS phase). We shall provide here arguments which agree with [23] and are based on the introduction of a Chern-Simons term into to compact QED description of the non-frustrated Heisenberg models.

We consider the π-flux state approach introduced by Affleck and Marston [3,4]. In this description it was shown that the flux through a plaquette formed by four spin sites must be equal to multiples of π in order to satisfy the projection properties of the loop operator [5]. The flux can be strictly fixed to $k\pi$ where k is an integer by means of a Chern-Simons (CS) term. We introduce such a term here in order to fix the flux and assure the conservation of the $SU(2)$ symmetry of the quantum spin system.

It is well known that in compact Maxwell theory Dirac magnetic monopoles (instantons) in (2+1) dimensions lead to confinement of test particles [13]. The question now arises about the effects produced by the introduction of a Chern-Simons term in the compact π-flux description of the Heisenberg interaction. We shall review well known results which lead to the conclusion that the flux through a plaquette controled by the CS term, screens only the magnetic field between spinons but does not affect the confining potential.

In the present approach the spin site-occupation is strictly fixed to one through the introduction of an imaginary chemical potential [6] avoiding the introduction of a Lagrange multiplier term [7].

The outline of the paper is as follows. In Section 2 we recall the main steps of the QED_3 formulation of the two-dimensional antiferromagnetic Heisenberg model. A justification for the implementation of the CS term is given and the modification induced by the presence of instantons is discussed. Section 3 deals with the derivation of the

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instanton action. In Section 4 the string tension of the potential between spinons is worked out.

2 Flux constraint in the presence of topological defects

Heisenberg quantum spin Hamiltonians of the type

$$H = -\frac{1}{2}\sum_{i,j} J_{ij}\boldsymbol{S}_i\boldsymbol{S}_j \tag{1}$$

with antiferromagnetic coupling $\{J_{ij}\} < 0$ can be mapped onto Fock space by means of the transformation $S_i^+ = f_{i,\uparrow}^\dagger f_{i,\downarrow}$, $S_i^- = f_{i,\downarrow}^\dagger f_{i,\uparrow}$ and $S_i^z = \frac{1}{2}(f_{i,\uparrow}^\dagger f_{i,\uparrow} - f_{i,\downarrow}^\dagger f_{i,\downarrow})$ where $\{f_{i,\sigma}^\dagger, f_{i,\sigma}\}$ are anticommuting fermion operators which create and annihilate spinons with $\sigma = \pm 1/2$. The projection onto Fock space is exact when the number of fermions per lattice site verifies $\sum_{\sigma=\pm 1/2} f_{i,\sigma}^\dagger f_{i,\sigma} = 1$. This is enforced here by using the Popov and Fedotov procedure [6,7] which introduces the imaginary chemical potential $\mu = i\pi/2\beta$ at temperature β^{-1}.

The Hamiltonian given by equation (1) is invariant under $SU(2)$ symmetry and also under the $U(1)$ gauge transformation

$$f_{i,\sigma} \to f_{i,\sigma}e^{ig\theta_i} \tag{2}$$

In $2d$ space the Heisenberg interaction can be written in terms of a π-flux mean-field Hamiltonian for which the mean-field flux ϕ_{mf} through a square plaquette of four spin sites is given by

$$\phi_{mf} = g\sum_{\langle ij\rangle\in\Box}(\theta_i - \theta_j) = \pi m$$

where θ_i is the gauge phase appearing in the gauge transformation (2) and m is an integer. The π-flux mean-field ansatz keeps the Hamiltonian (1) invariant under $SU(2)$ symmetry transformations. The dispersion relation of the π-flux mean-field Hamiltonian shows two independent nodal points. Near these nodal points the dispersion relation is linear with respect to the momentum vector [12].

In the neighbourhood of the nodal points and at low energy the Hamiltonian (1) can be rewritten in terms of a four-component Dirac spinon action in the continuum limit [8–10]. This action describes a spin liquid in (2+1) dimensions which includes the phase fluctuations $\delta\phi$ around the π-flux mean field phase ϕ_{mf}. It has been derived in [9] and reads

$$S_E = \int_0^\beta d\tau \int d^2r \left\{ -\frac{1}{2}a_\mu \left[(\Box\delta^{\mu\nu} + (1-\lambda)\partial^\mu\partial^\nu)\right] a_\nu + \sum_\sigma \bar{\psi}_{\boldsymbol{r}\sigma}\left[\gamma_\mu\left(\partial_\mu - iga_\mu\right)\right]\psi_{\boldsymbol{r}\sigma}\right\}. \tag{3}$$

In the following we consider the zero temperature limit $\beta \to \infty$. Here $a_\mu = \partial_\mu\theta$ is a gauge field generated from the $U(1)$ symmetry invariance of S_E when $\psi \to e^{ig\theta}\psi$. The bi-spinor Dirac spinon field ψ is defined by

$$\psi_{\boldsymbol{k}\sigma} = \begin{pmatrix} f_{1a,\boldsymbol{k}\sigma} \\ f_{1b,\boldsymbol{k}\sigma} \\ f_{2a\boldsymbol{k}\sigma} \\ f_{2b\boldsymbol{k}\sigma} \end{pmatrix}$$

where $f_{1,\boldsymbol{k},\sigma}^\dagger$ and $f_{1,\boldsymbol{k},\sigma}$ ($f_{2,\boldsymbol{k},\sigma}^\dagger$ and $f_{2,\boldsymbol{k},\sigma}$) are fermion creation and annihilation operators which act near the nodal points $(\frac{\pi}{2},\frac{\pi}{2})$ $((-\frac{\pi}{2},\frac{\pi}{2}))$ of the momentum $\boldsymbol{k}$. Indices a and b characterize the rotated operators

$$\begin{cases} f_{a,\boldsymbol{k},\sigma} = \frac{1}{\sqrt{2}}\left(f_{\boldsymbol{k},\sigma} + f_{\boldsymbol{k}+\boldsymbol{\pi},\sigma}\right) \\ f_{b,\boldsymbol{k},\sigma} = \frac{1}{\sqrt{2}}\left(f_{\boldsymbol{k},\sigma} - f_{\boldsymbol{k}+\boldsymbol{\pi},\sigma}\right). \end{cases}$$

The constant g in (3) is the coupling strength between a_μ and ψ. The first term corresponds to the "Maxwell" term $-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$ where $\mathcal{F}^{\mu\nu} = \partial^\mu a_\nu - \partial^\nu a_\mu$, λ is the parameter of the Faddeev-Popov gauge fixing term $-\lambda\left(\partial^\mu a_\mu\right)^2$ [11], $\delta^{\mu\nu}$ the Kronecker δ, $\Box = \partial_\tau^2 + \boldsymbol{\nabla}^2$ the Laplacian in Euclidean space-time. This form of the action originates from a shift of the imaginary time derivation $\partial_\tau \to \partial_\tau + \mu$ where μ is the imaginary chemical potential introduced above. It leads to a new definition of the Matsubara frequencies of the fermion fields [6] ψ which then read $\widetilde{\omega}_{F,n} = \omega_{F,n} - \mu/i = \frac{2\pi}{\beta}(n + 1/4)$.

Fluctuations of the flux around the π-flux mean-field are constrained by means of symmetry considerations on the loop operator $\Pi = f_{\boldsymbol{i}}^\dagger f_{\boldsymbol{i}+\boldsymbol{e}_x} f_{\boldsymbol{i}+\boldsymbol{e}_x}^\dagger f_{\boldsymbol{i}+\boldsymbol{e}_x+\boldsymbol{e}_y} f_{\boldsymbol{i}+\boldsymbol{e}_x+\boldsymbol{e}_y}^\dagger f_{\boldsymbol{i}+\boldsymbol{e}_y}^\dagger f_{\boldsymbol{i}}$. As shown by Marston [5], only gauge configurations of the flux states belonging to Z_2 symmetry ($\pm\pi$) are allowed. Hence the flux through a four-site plaquette is restricted to $\phi_\Box = \phi^{mf} + \delta\phi = \{0, \pm\pi\}\,(\text{mod } 2\pi)$. This was derived in the following way [5]. The loop operator verifies $\Pi^3 = \Pi$. Defining two quantum states $|u\rangle = \Pi^2|\varphi\rangle$ and $|v\rangle = (1 - \Pi^2)|\varphi\rangle$ where $|\varphi\rangle = |u\rangle + |v\rangle$ is a general quantum state it is easy to see that $\langle v|\Pi|v\rangle = 0$ and $\Pi^2|u\rangle = |u\rangle$. From the last equality one deduces that $|u\rangle$ can be decomposed into the eigenstates of Π with eigenvalues ± 1. The loop operator can also be rewritten as $\Pi = |\Pi|e^{i\phi_\Box}$ where $\phi_\Box$ is the total flux through the plaquette. In order to guarantee the properties of Π the total flux through the plaquette has to verify $\phi_\Box = \pi k$ where k is an integer. Other values are thus "forbidden" gauge configurations.

In order to remove these configurations ($\phi_\Box \neq \pm\pi$) in the case of the Heisenberg antiferromagnet a CS term is introduced in the QED_3 action in order to fix the total flux through the plaquette. This leads to the Maxwell-Chern-Simons (MCS) action in Euclidean space

$$S_E = \int_0^\beta d\tau \int d^2r \Big\{ -\frac{1}{2}a_\mu \Big[(\Box\delta^{\mu\nu} + (1-\lambda)\partial^\mu\partial^\nu) + i\kappa\varepsilon^{\mu\rho\nu}\partial_\rho \Big] a_\nu + \sum_\sigma \bar{\psi}_{\boldsymbol{r}\sigma}\left[\gamma_\mu\left(\partial_\mu - iga_\mu\right)\right]\psi_{\boldsymbol{r}\sigma} \Big\} \tag{4}$$

where g couples the spinon field to the gauge field and κ is the CS coefficient. Under normal conditions the CS contribution breaks parity and time-reversal invariance. However the CS coefficient κ can be chosen in such a way that the variation of the CS action under a gauge transformation can be an integer multiple of 2π. Indeed under a gauge transformation $a_\mu \to a_\mu + \partial_\mu \Lambda$ the variation of the CS action can be rewritten $\delta S_{CS} = \kappa \int d\Sigma_\mu \left(\Lambda \bar{\mathcal{F}}_\mu\right)$ where $\bar{\mathcal{F}}_\mu \equiv \frac{1}{2}\epsilon_\mu{}^{\alpha\lambda}\mathcal{F}_{\alpha\lambda}$. Specializing to the gauge transformation $\Lambda = (2\pi n/\beta)\tau$ where n is an integer and only different from zero inside a plaquette the variation of the CS action reads $\delta S_{CS} = \kappa 2\pi n \int d^2 r \bar{\mathcal{F}}_0$. The integration is simply equal to the flux passing through a plaquette and the variation of the CS action is equal to $\kappa 2\pi n \phi_\Box$. Since the flux must be a multiple of π the CS coefficient κ can always be chosen such that $\delta S_{CS} = 2\pi m$ where m is an integer [5]. Under this condition the variation of the CS action no longer contributes to the path integral and the effects of P and T symmetry breaking are avoided [5]. The magnetic field $\mathcal{B}$ through a plaquette is related to the flux constraint $\phi_\Box = \pi(\text{mod } 2\pi)$ and can be fixed through the CS action with such a specific coefficient κ [5,12]. Moreover states of the spin system for which the flux through plaquettes is a multiple of π are all equivalent and connected through gauge transformations. The variation of the CS action under such gauge transformations does not contribute to the path integral as mentioned before.

Instanton generation from the compactness of the gauge field connects these different spin states with fluxoids equal to 2π. In the compact QED_3 description of the π-flux mean field action the symmetry ($\Pi^3 = \Pi$) of the loop operator remains unbroken. The flux through a plaquette $\phi_\Box$ has to be fixed to multiples of π even in presence of instantons. The instantons introduce a flux through the plaquette equal to $\phi_{inst} = 2\pi q$ where the integer q is the total *winding* charge of the instantons in the plaquette. The flux through a plaquette is $\phi_\Box = \phi^0 + \phi_{inst}$ where ϕ^0 is the flux without instantons. It is therefore clear that ϕ^0 has to be fixed to multiples of π to ensure the symmetry of Π. Hence the Chern-Simons term is introduced to control the fluctuations of ϕ^0 but it does not affect the fluctuations of the instanton density.

The compact Maxwell-Chern-Simons (MCS) action for the Heisenberg model then reads

$$S_E^{compact} = \int_0^\beta d\tau \int d^2 r \Big\{ -\frac{1}{2} a_\mu \left[(\Box \delta^{\mu\nu} + (1-\lambda)\partial^\mu \partial^\nu) \right] a_\nu + \sum_\sigma \bar{\psi}_{\boldsymbol{r}\sigma} \left[\gamma_\mu \left(\partial_\mu - i g a_\mu \right) \right] \psi_{\boldsymbol{r}\sigma} \Big\}_{\mathcal{F}_{\mu\nu} \to \widetilde{\mathcal{F}}_{\mu\nu}} + \int_0^\beta d\tau \int d^2 r \frac{1}{2} a_\mu \Big[- i\kappa \varepsilon^{\mu\rho\nu} \partial_\rho \Big] a_\nu \qquad (5)$$

where the compact version of the Maxwell and spinon action is generated through the transformation

$$\mathcal{F}_{\mu\nu} \to \widetilde{\mathcal{F}}_{\mu\nu} = \mathcal{F}_{\mu\nu} - 2\pi n_{x,\mu\nu}$$

where $\mathcal{F}$ is the electromagnetic tensor defined above in the absence of instantons. In this transformation, $n_{x,\mu\nu} = \varepsilon_{\mu\nu\gamma}\partial_\gamma \varphi_x$ where φ_x is the scalar potential generated by the instanton charge q_x through the Poisson equation $\Delta_{x,x'}\varphi_{x'} = q_x$ where q_x is an integer [13].

3 Instanton action with flux-controlled spinon field

Integrating out the matter field ψ the MCS action (4) leads to the definition of the gauge field propagator at zero temperature [12,14]

$$\Delta_{E,\mu,\nu} = \frac{1}{k^2 \varepsilon_\kappa(k)} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - \frac{\kappa}{(k^2 + \Pi(k))} \varepsilon_{\mu\nu\rho} k_\rho \right) + \frac{k_\mu k_\nu}{\lambda (k^2)^2} \qquad (6)$$

where $\varepsilon_\kappa(k) = 1 + \frac{\Pi(k)}{k^2} + \frac{\kappa^2}{k^2 + \Pi(k)}$ is the dielectric function induced by the matter field and flux through plaquettes, κ is the CS coefficient as defined above and λ the Faddeev-Popov gauge fixing parameter. In this gauge field propagator $\Pi(k) = \alpha k$ is the polarization contribution at the one-loop approximation and $\alpha = 2g^2$ the coupling constant between the (pseudo)-electromagnetic field and the spinon field considered here as the fermionic matter field.

Instantons appear only in the Maxwell and spinon terms when $\mathcal{F}$ goes over to $\widetilde{\mathcal{F}}$

$$\int \frac{d^3 k}{(2\pi)^3} \varepsilon_{\kappa=0}(k) \frac{1}{4} \widetilde{\mathcal{F}}_{\mu\nu}(k)^2 = \int \frac{d^3 k}{(2\pi)^3} \varepsilon_{\kappa=0}(k) \frac{1}{4} \left(\mathcal{F}_{\mu\nu}(k) - 2\pi n_{k,\mu\nu} \right)^2$$

which leads to the partition function of the gauge field a_μ

$$\mathcal{Z} = \mathcal{Z}_{(0)} \times \mathcal{Z}_{inst}$$

where $\mathcal{Z}_{(0)}$ and $\mathcal{Z}_{inst}$ are respectively the bare electromagnetic and the instanton contribution to the partition function. One obtains

$$\mathcal{Z}_{(0)} = \int \mathcal{D} a_\mu e^{-\frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} a_\mu \Delta^{-1}_{\mu\nu} a_\nu}.$$

The topological defects created by instantons through the compactification lead to $\mathcal{Z}_{inst}$ given by

$$\mathcal{Z}_{inst} = \sum_{\{q_x\}} e^{-\int \frac{d^3 k}{(2\pi)^3} 4\pi^2 \varphi_{-k} \left(k^2 \varepsilon_{\kappa=0}(k) \right) \varphi_k} \qquad (7)$$

where φ_k is the Fourier transform of the scalar potential φ_x defined above and generated by the integer *winding* charges q_x over which the sum is performed in equation (7). The scalar potential φ_k is related to the instanton density $\rho_{inst}(x) = \sum_{x_a} q_a \delta(x - x_a)$ by the Poisson formula $\varphi_k = \frac{\rho_{inst}(k)}{k^2 \varepsilon_{\kappa=0}(k)}$ where the dielectric function $\varepsilon_\kappa(k)$ stems from the gauge field propagator (6).

The partition function (7) can be put in a functional integral form [13]. Performing a Hubbard-Stratonovich (HS) transformation on equation (7) with respect to the instanton charges q_a leads to

$$\mathcal{Z}_{inst} = \int \mathcal{D}\chi \left(e^{-\int \frac{d^3k}{(2\pi)^3}\chi(-k)\frac{k^2\varepsilon_{\kappa=0}(k)}{4\pi^2}\chi(k)} \right) \times \sum_N \sum_{\{q_a\}} \frac{\xi^N}{N!} \int \prod_{j=1}^{N} dx_j e^{i\sum_{\{x_a\}} q_a \chi(x_a)} \quad (8)$$

where ξ is the instanton fugacity which is related to the dielectric function through $\ln \xi = -\frac{1}{4\pi}\int \frac{d^3k}{(2\pi)^3}\frac{1}{\varepsilon_{\kappa=0}(k)k^2}$. The auxiliary field χ is generated by the HS transformation. Following references [13–15] we assume that $q_a = \pm 1$ are the only relevant instanton charges. Then

$$\mathcal{Z}_{inst} = \int \mathcal{D}\chi e^{-\frac{1}{(2\pi)^2}\int \frac{d^3k}{(2\pi)^3}\left(\chi(-k)k^2\varepsilon_{\kappa=0}(k)\chi(k)\right)} \times e^{\frac{M^2}{(2\pi)^2}\int d^3x \cos\chi(x)}. \quad (9)$$

In this last equation $M^2 = (2\pi)^2\xi$ induces a confining potential between two test particles [13]. At this point it is interesting to make a comparison of (9) with the classical instanton action given by Polyakov. In our case the matter field leads to the appearance of a dielectric function in the instanton partition function. This dielectric function induces modifications on the string tension between test particles as it will be shown in the next section. It affects also the dual field $H_\mu = \epsilon_{\mu\nu\rho}\mathcal{F}_{\nu\rho}$.

When instantons are present in the system the H-field is given by two terms, the bare electromagnetic field contribution

$$H_\mu^{(0)}(k) = \epsilon_{\mu\nu\rho}\mathcal{F}_{\nu\rho}(k) = \epsilon_{\mu\beta\gamma} i k_\beta a_\gamma(k)$$

and H_μ^{inst} which stems from the *magnetic* field created by instantons

$$H_\mu^{inst} = \frac{2\pi i k_\mu \rho_{inst}(k)}{k^2\varepsilon_{\kappa=0}(k)}.$$

The introduction of a matter field as well as a flux through plaquette controlled by a Chern-Simons coefficient κ induces a screening of the H-field as can be seen on the correlation function

$$\langle H_\mu(-k)H_\nu(k)\rangle = \frac{1}{\varepsilon_\kappa(k)}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + \frac{\kappa}{(k^2+\Pi(k))}\varepsilon_{\mu\nu\rho}k_\rho\right) + \frac{M^2}{\varepsilon_{\kappa=0}(k)\left(M^2 + k^2\varepsilon_{\kappa=0}(k)\right)}\frac{k_\mu k_\nu}{k^2}$$

where $H_\mu(k) = H_\mu^{(0)}(k) + H_\mu^{inst}(k)$. In the absence of topological defects $M = 0$ it is easy to see that the magnetic field is screened with a characteristic length $1/\kappa$ in agreement with [20]. In the case $M \neq 0$ the photons are massive [13] and the photon mass M is not affected by the Chern-Simons term. From this result we anticipate that the string tension between spinons will not be affected by the Chern-Simons term (i.e. the flux tied to each spinon and proportional to $1/\kappa$).

4 String tension between two test particles

The effective potential between two test particles can be obtained from the Wilson loop [17]. Given a loop contour C, the Wilson loop is a gauge invariant $W(C) = \langle e^{-\oint_C dx_\mu a_\mu(x)}\rangle$ and leads to the potential $V(R) = -\lim_{T\to\infty}\frac{1}{T}\ln W(C)$ [18] where R and T are the lengths of the loop C in the xy plane. If the potential is not confining the logarithm of the Wilson loop is proportional to $T + R$, this is the so called perimeter law, and if the potential is confining it leads to the area law $\ln W(C) \propto RT$.

We shall now show that when a matter field is present the Wilson loop follows the area law but the string tension is reduced by screening effects.

The Wilson loop operator can be rewritten $W(C) = \langle e^{i\int H_\mu dS_\mu}\rangle = \langle e^{i\int H_\mu^0 dS_\mu}\rangle_{\mathcal{Z}_{(0)}} \times \langle e^{i\int H_\mu^{inst} dS_\mu}\rangle_{\mathcal{Z}_{inst}}$ where the H-field has been separated into the bare gauge field and the instanton H-field contributions. The average over the bare gauge field leads to the screened [12,19] Coulomb interaction and will be disregarded here. The second average leads to the instanton confining potential which is of interest here. The Wilson loop with respect to the instanton action reads

$$\begin{aligned} W_{inst}(C) &= \langle e^{-\oint_C dx_\mu a_\mu(x)}\rangle_{inst} \\ &= \langle e^{-\int_{x\in C} dS_\mu H_\mu^{inst}(x)}\rangle_{inst} \\ &= \int \mathcal{D}\chi e^{-\frac{1}{(2\pi)^2}\int \frac{d^3k}{(2\pi)^3}\left([\chi_{-k}-\eta_{-k}]k^2\varepsilon_{\kappa=0}(k)[\chi_k-\eta_k]\right)} \\ &\quad \times e^{\frac{M^2}{(2\pi)^2}\int d^3x \cos\chi(x)} \end{aligned} \quad (10)$$

where $\langle\ldots\rangle_{inst}$ stands as an average induced by the instanton partition function $\mathcal{Z}_{inst}$. In equation (10) $\eta(-k) = \int dS_x \frac{2\pi i k_\mu}{k^2\varepsilon_{\kappa=0}(k)} e^{ik.x}$ and $H_\mu^{inst}(k)$ is given in Section 3.

The Wilson loop can be approximated by the classical solution χ_{cl} obtained by a saddle-point method on the functional integral (10) and in the limit $R, T \to \infty$ one gets the classical solution

$$\chi_{cl}(k) = \frac{-2\pi i k_z e^{-ik_z z}.(2\pi)^2\delta(k_x)\delta(k_y)}{(k^2\varepsilon_{\kappa=0}(k) + M^2)}. \quad (11)$$

Here we assumed that χ_{cl} is sufficiently small so that $\cos\chi_{cl} \propto 1 - \frac{1}{2}\chi_{cl}^2$ leading to equation (12). The introduction of $\chi_{cl}(k)$ into (10) leads to

$$W_{inst}(C) = e^{-g^2 RT(-\partial_z^2)\left(F\left[\frac{1}{k^2\varepsilon_{\kappa=0}(k)}\right]_z - F\left[\frac{1}{k^2\varepsilon_{\kappa=0}(k)+M^2}\right]_z\right)} \quad (12)$$

where $F\left[f(k)\right]_z = \int \frac{dk}{(2\pi)}e^{ikz}f(k)$ is the Fourier transform with respect to the variable z (see Appendix A).

In the strong coupling limit $\alpha k \gg k^2$ and $k^2\varepsilon_{\kappa=0}(k) = k\alpha$. In this case the string tension reads

$$\sigma_s \simeq \frac{g^2 M^2}{\alpha^2}.$$

It was shown in [21] that the absence of a matter field leads to a string tension $\sigma = Mg^2/4\pi$. One sees that a finite matter field coupled to the electromagnetic field through α affects the string tension and screens the coupling between test particles [15]. The Chern-Simons term no longer affects the confining mass M in this treatment of the non-frustrated Heisenberg model. Each spinon is tied to a flux proportional to $1/\kappa$ [22]. However the instanton flux is independent of the symmetries underlying the Heisenberg model, in other words it is not controlled by κ. The topological charges of instantons are not altered by the spinon flux $1/\kappa$. This leads to a string tension unaffected by the CS flux but screened by matter field. The spinons remain confined and lead to the absence of paramagnetic phase in the non-frustrated Heisenberg model at zero temperature, with respect to this treatment.

5 Conclusion

We mapped a two dimensional Heisenberg Hamiltonian on an (2+1)-dimensional compact quantum electrodynamic Lagrangian with a Maxwell-Chern-Simons term at zero temperature. Here the spin site-occupation constraint is rigorously fixed by means of an imaginary chemical potential term[6,7] which avoids the use a Lagrange multiplier constraint.

By symmetry consideration on a loop operator formed with the fermion operator describing the spin around a plaquette it is shown how a Chern-Simons term enter the QED_3 description of the Heisenberg interaction. The flux through the plaquette is fixed to multiples of π in order to enforce $SU(2)$ symmetry on the Heisenberg interaction. The Chern-Simons action is introduced after taking the compact version of the Maxwell-Spinon action in order to control the flux through plaquettes formed by the spins.

We worked out the string tension of the confining potential which acts between the spinons and showed that the CS term induces a screening effect on the magnetic field. The confining potential between spinons is affected by the matter field alone.

In conclusion we addressed the question about the possibility of controlling the deconfinement of spinons through flux affixed to them and proportional to the inverse of the Chern-Simons parameter κ. The confining string tension is not affected by the CS parameter even though this is the case for the magnetic field. Our treatment agrees with the fact that for an unfrustrated Heisenberg model the spinon would not be in a deconfined phase [23]. At zero temperature non-frustrated Heisenberg systems should not present a paramagnetic phase.

A better treatment would be to take the matter-screened instanton flux ϕ_{inst} into account and fix to multiples of π the total flux through plaquette $\phi_\square$. This could possibly lead to consider instanton configuration with winding charge greater than one as well as a string tension depending on the spinon flux $1/\kappa$.

Appendix A

In the case of strong coupling $\alpha k \gg k^2$ the dielectric function reads $k^2\varepsilon_\kappa(k) = k\left(\alpha + \kappa^2/\alpha\right)$ and one gets

$$\begin{aligned} F\left[\frac{1}{k^2\varepsilon_{\kappa=0}(k)}\right]_z &= \frac{1}{\alpha} i\theta(z) \\ F\left[\frac{1}{k^2\varepsilon_{\kappa=0}(k)+M^2}\right]_z &= \frac{1}{\alpha} i\theta(z) e^{-i\frac{M^2}{\alpha}z}. \end{aligned}$$

Here we choose to define the step function $\theta(z)$ as

$$\theta(z) = \left| \begin{array}{ll} 1 & z > 0 \\ 0 & z \leq 0. \end{array} \right.$$

References

1. P.A. Lee, N. Nagaosa, X.-G. Wen, Rev. Mod. Phys. **78**, 17 (2006)
2. T. Senthil, A. Vishwanath, L. Balents, S. Sachdev, M.P.A. Fisher, Science **303**, 1490 (2004)
3. I. Affleck, J.B. Marston, Phys. Rev. B **37**, 3774 (1988)
4. J.B. Marston, I. Affleck, Phys. Rev. B **39**, 11538 (1989)
5. J. Brad Marston, Phys. Rev. Lett. **61**, 1914 (1988)
6. V.N. Popov, S.A. Fedotov, Sov. Phys. JETP **67**, 535 (1988)
7. R. Dillenschneider, J. Richert, Phys. Rev. B **73**, 024409 (2006)
8. P. Ghaemi, T. Senthil, Phys. Rev. B **73**, 054415 (2006)
9. R. Dillenschneider, J. Richert, Phys. Rev. B **73**, 224443 (2006)
10. T. Morinari, e-print `arXiv:cond-mat/0508251` (2005)
11. C. Itsykson, J.-B. Zuber, Quantum Field Theory, McGraw-Hill, (1986)
12. R. Dillenschneider, J. Richert, Phys. Rev. B **74**, 144404 (2006)
13. A.M. Polyakov, *Gauge Fields and Strings* (Harwood Academic Publishers, 1987)
14. F.S. Nogueira, H. Kleinert, Phys. Rev. Lett. **95**, 176406 (2005)
15. Z. Nazario, D.I. Santiago, e-print `arXiv:cond-mat/0606386` (2006)
16. S. Deser, R. Jackiw, S. Templeton, Ann. Phys. (NY) **140**, 372 (1980)
17. K.G. Wilson, Phys. Rev. D **10**, 2445 (1974)
18. N. Nagaosa, *Quantum field theory in condensed matter physics* (Springer, 1999)
19. M. Lüscher, Nucl. Phys. B **180**, 317 (1981)
20. M.C. Diamantini, P. Sodano, Phys. Rev. Lett. **71**, 1969 (1993)
21. T. Yazawa, T. Suzuki, JHEP **4**, 026 (2001)
22. G.V. Dunne, *Aspects of a Chern-Simons theory* (`hep-th/9902115`, 1999)
23. F.S. Nogueira, H. Kleinert, e-print `arXiv:cond-mat/0705.3541` (2007)